

# General Certificate of Education 

## Mathematics 6360

## MM03

Mechanics 3

## Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Key to mark scheme and abbreviations used in marking

| M | mark is for method |  |  |
| :--- | :--- | :--- | :--- |
| m or dM | mark is dependent on one or more M marks and is for method |  |  |
| A | mark is dependent on M or m marks and is for accuracy |  |  |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) | $\begin{aligned} & \mathrm{MLT}^{-2}=\frac{[G] \mathrm{MM}}{\mathrm{~L}^{2}} \\ & {[G]=\mathrm{L}^{3} \mathrm{M}^{-1} \mathrm{~T}^{-2}} \end{aligned}$ | $\begin{gathered} \hline \text { M1 } \\ \text { A1 } \\ \text { A1F } \end{gathered}$ | 3 |  |
| (b) | $\begin{aligned} & t=k m^{\alpha} R^{\beta} G^{\gamma} \\ & \mathrm{T}=\mathrm{M}^{\alpha} \mathrm{L}^{\beta} \mathrm{M}^{-\gamma} \mathrm{L}^{3 \gamma} \mathrm{~T}^{-2 \gamma} \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1F } \end{gathered}$ |  | $\mathrm{L}, \mathrm{M}, \mathrm{T}$ for $G$ are needed to gain M1 |
|  | $\begin{aligned} & -2 \gamma=1 \Rightarrow \gamma=-\frac{1}{2} \\ & \alpha-\gamma=0 \Rightarrow \alpha=-\frac{1}{2} \\ & \beta+3 \gamma=0 \Rightarrow \beta=\frac{3}{2} \end{aligned}$ | m1 <br> m1 <br> A1F | 5 | Getting 3 equations <br> Solution <br> Finding $\alpha, \beta, \gamma$ |
|  | Total |  | 8 |  |

MM03 (cont)

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 2 (a) | $\begin{aligned} { }_{B} \mathbf{v}_{A} & =\mathbf{v}_{A}-\mathbf{v}_{B} \\ & =(20 \mathbf{i}-10 \mathbf{j}+20 \mathbf{k})-(30 \mathbf{i}+10 \mathbf{j}+10 \mathbf{k}) \\ & =-10 \mathbf{i}-20 \mathbf{j}+10 \mathbf{k} \end{aligned}$ | M1A1 | 2 | Simplification not necessary |
| (b) | $\begin{aligned} { }_{B} \mathbf{r}_{0 A}=(8000 \mathbf{i}+ & 1500 \mathbf{j}+3000 \mathbf{k}) \\ & -(2000 \mathbf{i}+500 \mathbf{j}+1000 \mathbf{k}) \end{aligned}$ | M1 |  |  |
|  | $\begin{aligned} &{ }_{\mathrm{B}} \mathbf{r}_{\mathrm{A}}=(6000 \mathbf{i}+1000 \mathbf{j}+2000 \mathbf{k}) \\ &+(-10 \mathbf{i}-20 \mathbf{j}+10 \mathbf{k}) t \\ &{ }_{\mathrm{B}} \mathbf{r}_{\mathrm{A}}=(6000-10 t) \mathbf{i}+(1000-20 t) \mathbf{j} \\ &+(2000+10 t) \mathbf{k} \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1F } \end{gathered}$ | 3 | Simplification not necessary |
| (c) | $\begin{aligned} \left\|\mathbf{B}_{A}\right\|^{2}=(6000-10 t)^{2}+ & (1000-20 t)^{2} \\ & +(2000+10 t)^{2} \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1F } \end{gathered}$ |  |  |
|  | The helicopters are closest when $\left\|{ }_{B} \mathbf{r}_{A}\right\|^{2}$ is minimum. $\begin{aligned} & y=(6000-10 t)^{2}+(1000-20 t)^{2} \\ & +(2000+10 t)^{2} \\ & \frac{\mathrm{~d} y}{\mathrm{~d} t}=2(-10)(6000-10 t) \\ & +2(-20)(1000-20 t) \\ & +2(10)(2000+10 t)=0 \end{aligned}$ | $\begin{gathered} \mathrm{ml} \\ \mathrm{AlF} \end{gathered}$ |  |  |
|  | $t=100$ <br> Alternative: $\left(\begin{array}{l} 6000-10 t \\ 1000-20 t \\ 2000+10 t \end{array}\right) \cdot\left(\begin{array}{r} -10 \\ -20 \\ 10 \end{array}\right)=0$ | A1F <br> (M1) <br> (A1F) | 5 |  |
|  | $\begin{aligned} -60000+100 t- & 20000+400 t \\ & +20000+100 t=0 \end{aligned}$ | $\begin{gathered} (\mathrm{m} 1) \\ (\mathrm{A} 1 \mathrm{~F}) \end{gathered}$ |  |  |
|  | $\begin{aligned} & 600 t=60000 \\ & t=100 \end{aligned}$ | (A1F) | (5) |  |
|  | Total |  | 10 |  |

MM03 (cont)


MM03 (cont)

\begin{tabular}{|c|c|c|c|c|}
\hline Q \& Solution \& Marks \& Total \& Comments <br>
\hline 5(a) \& $$
y=u t \sin \alpha-\frac{1}{2} g t^{2}
$$ \& $$
\begin{aligned}
& \hline \text { M1 } \\
& \text { A1 }
\end{aligned}
$$ \& \& <br>
\hline \& $x=u t \cos \alpha$ \& M1 \& \& <br>
\hline \& $$
t=\frac{x}{u \cos \alpha}
$$ \& A1 \& \& <br>
\hline \& $$
\begin{aligned}
& y=u\left(\frac{x}{u \cos \alpha}\right) \sin \alpha-\frac{1}{2} g\left(\frac{x}{u \cos \alpha}\right)^{2} \\
& y=x \tan \alpha-\frac{g x^{2}}{u^{2} \cos ^{2} \alpha} \\
& y=x \tan \alpha-\frac{g x^{2}}{2 u^{2}}\left(1+\tan ^{2} \alpha\right)
\end{aligned}
$$ \& M1

A1 \& 6 \& Answer given <br>

\hline (b)(i) \& \[
$$
\begin{aligned}
& 1=R \tan \alpha-\frac{10 R^{2}}{2 u^{2}}\left(1+\tan ^{2} \alpha\right) \\
& 5 R^{2} \tan ^{2} \alpha-u^{2} R \tan \alpha+5 R^{2}+u^{2}=0
\end{aligned}
$$

\] \& | M1 |
| :--- |
| A1 | \& 2 \& Answer given <br>

\hline (ii) \& For real solutions of the quadratic :

\[
$$
\begin{aligned}
& u^{4} R^{2}-20 R^{2}\left(5 R^{2}+u^{2}\right) \geq 0 \\
& R^{2} \leq \frac{u^{4}-20 u^{2}}{100} \\
& R^{2} \leq \frac{u^{2}\left(u^{2}-20\right)}{100}
\end{aligned}
$$

\] \& | M1 |
| :--- |
| A1 | \& 2 \& Answer given <br>

\hline (iii) \& \[
$$
\begin{aligned}
& 5^{2} \leq \frac{u^{2}\left(u^{2}-20\right)}{100} \\
& u^{4}-20 u^{2}-2500 \geq 0 \\
& u_{\text {min }}{ }^{2}=61.0 \quad(\text { or } 10+\sqrt{2600}) \\
& u_{\text {min }}=7.81
\end{aligned}
$$

\] \& | M1 |
| :--- |
| A1 |
| A1F | \& 3 \& | Condone equation |
| :--- |
| 3 sf required | <br>

\hline \& Total \& \& 13 \& <br>
\hline
\end{tabular}

MM03 (cont)

\begin{tabular}{|c|c|c|c|c|}
\hline Q \& Solution \& Marks \& Total \& Comments \\
\hline 6(a) \& \begin{tabular}{l}
Before: \\
After: \\
Con. of Mom. along the line of centres: \(m u \cos 30^{\circ}=m v_{A}+m v_{B}\)
\[
\begin{equation*}
v_{A}+v_{B}=\frac{\sqrt{3}}{2} u \tag{1}
\end{equation*}
\] \\
Newton's experimental law :
\[
\begin{align*}
\& e=\frac{v_{B}-v_{A}}{u \cos 30^{\circ}-0} \\
\& v_{B}-v_{A}=\frac{\sqrt{3}}{2} u e \tag{2}
\end{align*}
\] \\
Solving (1) and (2) :
\[
\begin{aligned}
\& v_{B}=\frac{\sqrt{3}}{4} u(1+e) \\
\& \perp \quad u \sin 30^{\circ}=\frac{1}{2} u \\
\& \| \quad v_{A}=\frac{\sqrt{3}}{2} u-\frac{\sqrt{3}}{4} u(1+e) \\
\& v_{A}=\frac{\sqrt{3}}{4} u(1-e) \\
\& \alpha=\tan ^{-1} \frac{\frac{1}{2}}{\frac{\sqrt{3}}{4}} u\left(1-\frac{2}{3}\right) \\
\& \alpha=\tan ^{-1} \frac{6}{\sqrt{3}} \\
\& \alpha=74^{\circ}
\end{aligned}
\]
\end{tabular} \& \begin{tabular}{l}
M1 \\
A1 \\
M1 \\
A1 \\
A1 \\
B1 \\
M1 \\
A1F \\
M1 \\
A1F \\
A1F
\end{tabular} \& 5
3
3

3 \& | Answer given |
| :--- |
| $u \sin 30$ accepted |
| Simplification not needed |
| To the nearest degree required | <br>

\hline \& Total \& \& 11 \& <br>
\hline
\end{tabular}

MM03 (cont)

\begin{tabular}{|c|c|c|c|c|}
\hline Q \& Solution \& Marks \& Total \& Comments \\
\hline 7(a) \& \begin{tabular}{l}
\[
\begin{aligned}
\& y=u t \sin \theta-\frac{1}{2} g t^{2} \cos \theta \\
\& y=0 \quad \Rightarrow \quad t=\frac{2 u \sin \theta}{g \cos \alpha} \\
\& x=u t \cos \theta-\frac{1}{2} g t^{2} \sin \alpha \\
\& R=u \frac{2 u \sin \theta}{g \cos \alpha} \cos \theta-\frac{1}{2} g\left(\frac{2 u \sin \theta}{g \cos \alpha}\right)^{2} \sin \alpha \\
\& R=\frac{2 u^{2} \sin \theta \cos (\theta+\alpha)}{g \cos ^{2} \alpha} \\
\& R=\frac{2 u^{2} \times \frac{1}{2}[\sin (2 \theta+\alpha)+\sin (-\alpha)]}{g \cos ^{2} \alpha}
\end{aligned}
\] \\
R is maximum when \(\sin (2 \theta+\alpha)=1\)
\[
\begin{aligned}
\& \text { or } 2 \theta+\alpha=\frac{\pi}{2} \\
\& \therefore \quad \theta=\frac{\pi}{4}-\frac{\alpha}{2} \\
\& y=0 \Rightarrow t=\frac{2 u \sin \theta}{g \cos \alpha} \\
\& \dot{x}=0 \Rightarrow t=\frac{u \cos \theta}{g \sin \alpha} \\
\& \frac{2 u \sin \theta}{g \cos \alpha}=\frac{u \cos \theta}{g \sin \alpha} \\
\& 2 \tan \theta=\cot \alpha
\end{aligned}
\]
\end{tabular} \& \begin{tabular}{l}
M1A1 \\
A1F \\
M1A1 \\
M1 \\
m1 \\
A1 \\
B1 \\
M1 \\
A1 \\
M1 \\
A2,1 \\
A1
\end{tabular} \& 8

3

4 \& | Dependent on M1s |
| :--- |
| Answer given |
| Answer given |
| For using $y=0$ and $\dot{x}=0$ |
| A2 for both correct |
| Answer given |
| N.B. A problem arose which ultimately affected the marking of part 7(c). |
| Please see the Report on the Examination for details. | <br>

\hline \& Total \& \& 15 \& <br>
\hline \& TOTAL \& \& 75 \& <br>
\hline
\end{tabular}


[^0]:    Set and published by the Assessment and Qualifications Alliance.

